Inference for SPDEs and related topics

Prague–Berlin workshop

BOOK OF ABSTRACTS

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The LAN property for local measurements

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Abstract

In this talk we consider the parametric estimation of the diffusivity in a linear stochastic heat equation with additive noise. Following the statistical setup in [1], we observe M local measurement processes at different locations. Each measurement corresponds to the spatial average of the process relative to a known point spread kernel function with support diameter δ. As $\delta \to 0$ and $M \to \infty$, we discuss the LAN (local asymptotic normality) expansion of the likelihood process. The proof is inspired by the classical Kalman-Bucy filter.

References

[1] R. Altmeyer, A. Tiepner and M. Wahl. Optimal parameter estimation for linear SPDEs from multiple measurements. *Annals of Statistics*, to appear.

Viscosities estimation for Stochastic Primitive Equations

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Abstract
We develop several estimators for the anisotropic viscosities in the Stochastic Primitive Equations (SPE), within the spectral approach to statistical analysis for SPDEs. The focus is on establishing consistency and asymptotic normality of these estimators. We consider a torus domain and treat strong, pathwise solutions in the presence of additive white noise (in time). We show that the analysis for estimating horizontal and vertical viscosities differs due to the unique structure of the SPE and the fact that both parameters of interest are adjacent to the highest-order derivative. In the second part of the talk, we will discuss some new results and approaches to statistical analysis for SPDEs and SDEs using tools from deep learning.

Besov-Orlicz regularity of stochastic processes

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Abstract

The talk will be devoted to Besov-Orlicz regularity of sample paths of, possibly non-Gaussian, stochastic processes. The attention will be paid to processes that are represented by multiple Wiener-Itô integrals of order $n \in \mathbb{N}$ with Hölder continuous kernels of order $\alpha \in (0,1)$. We will give sufficient conditions for such processes to have paths in the exponential Besov-Orlicz space

$$B^{\alpha}_{\Phi_{2/n},\infty}(0,T) \qquad \text{with} \qquad \Phi_{2/n}(x) \sim \mathrm{e}^{x^{2/n}} - 1.$$

These results provide an extension of what is known for scalar Gaussian processes (such as fractional Brownian motions) to stochastic processes in an arbitrary finite Wiener chaos (such as Rosenblatt or higher-order fractionally filtered Hermite processes).

Acknowledgment

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References

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Bayesian inference in semi-linear SPDEs using spatial information

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Abstract

We consider the Bayesian non-parametric estimation of the reaction term in a semi-linear parabolic SPDE. Consistency is achieved by making use of the spatial ergodicity of the SPDE while the time horizon is fixed. The analysis of the estimation error requires new concentration results for spatial averages of transformation of the SPDE, which are based the combination of the Clark-Ocone formula with bounds on the marginal densities. The general methodology is exemplified in the asymptotic regime, where the diffusivity level and the noise level of the SPDE tend to zero in a realistic coupling.

Differential equations driven by exponential Besov-Orlicz paths

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Abstract

Stochastic processes (for example Brownian motion), that can be added to differential equations to account for non-systematic error or uncertainty in the model, typically have nowhere differentiable sample paths. Thus, a notion of an integral (and therefore that of a solution to the equation) has to be constructed in a different way than path by path. Rough path theory is, in a certain sense, a way to overcome this problem.
The standard rough path theory is built for drivers with Hölder continuous paths and therefore it gets Hölder continuity of the paths of the solution in exchange. However, there are more suitable function spaces for all the canonical examples of drivers in stochastic differential equations (for example exponential Besov-Orlicz spaces). It turns out that if we restrict ourselves to such spaces, the solution lives in these, smaller, spaces as well.

Acknowledgment

Research supported by the Charles University Grant Agency project no. 178823.

References

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Parameter estimation for the stochastic heat equation with multiplicative noise from local measurements

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Abstract

We propose the stochastic heat equation driven by the multiplicative space-time white noise and we introduce several estimators to the diffusivity parameter that are inspired by the maximum likelihood estimator from [3]. Since our observation scheme relies on local measurements, the asymptotics of the estimators is pursued with a fixed time horizon and with the spatial resolution of the observations tending to zero. A stable central limit theorem from [4] shows that our proposed estimators are strictly consistent and asymptotically mixed normal. The performance of the estimators is demonstrated in numerical simulations.

Acknowledgment

This research has been funded by Deutsche Forschungsgemeinschaft (DFG) - SFB1294/2-318763901.

References

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[2] Altmeyer, R., Cialenco, I., Pasemann, G., (2023): Parameter estimation for semilinear SPDEs from local measurements. *Bernoulli* **29**(3), 2035–2061.

[3] Altmeyer, R., Reiß, M., (2021): Nonparametric estimation for linear SPDEs from local measurements. *Annals of Applied Probability* **31**(1), 1–38.

[4] Jacod, J., Shiryaev, A. N., (2013): Limit theorems for stochastic processes, second edition. *In: A Series of Comprehensive Studies in Mathematics* **288**.

[5] Janák, J., Reiß, M., (2024): Parameter estimation for the stochastic heat equation with multiplicative noise from local measurements. *Stochastic Processes and their Applications* **175**.

Estimating Hurst parameter for stochastic heat equation – spectral approach

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Abstract

In this talk, I will address the problem of estimation of Hurst parameter from spectral observations of a solution to a stochastic heat equation with an additive fractional noise. It turns out that the standard approach from scalar self-similar processes fails for higher spatial resolution. This is because the projections of solution are not truly self-similar. To overcome this, one has to perform parabolic rescaling (rescale jointly temporal and spatial/spectral dimensions) in accordance with parabolic self-similarity of the stochastic heat equation. Asymptotic properties of the corresponding estimator under different asymptotic regimes will be discussed.

Generative modelling and stochastic parametrisations for a rotating shallow water system

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Abstract

Stochastic parametrisation of small-scale processes are essential in the estimation of uncertainty when trying to represent systematic model errors which arise from small scale fluctuations (in e.g. weather and climate predictions). In a stochastic partial differential model, the noise can be calibrated in a way which is consistent with such subgrid scale parametrisations, using a principal component analysis (PCA). In this talk, we explain how the PCA technique can actually be replaced by a generative diffusion model technique - this allows us to avoid imposing additional constraints on the increments. We apply this methodology to a stochastic rotating shallow water model, using the elevation variable of the model as input data. This is joint work with Dan Crisan and Alexander Lobbe (Imperial College London) and it is based on [1].

References

[1] Crisan, D., Lang, O., Lobbe, A. (2024): Generative Modelling of Stochastic Rotation Shallow Water Noise. Preprint, https://arxiv.org/abs/2403.10578.

Numerical approximation of the stochastic total variation flow

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Abstract
The talk is based on a joint work with Ľ. Baňas (Bielefeld). We propose fully practical numerical schemes (conformal and non-conformal) for the simulation of the stochastic total variation flow (STVF). The approximation is based on a stable time-implicit finite element space-time approximation of a regularized STVF equation. The approximation also involves a finite dimensional discretization of the noise that makes the scheme fully implementable on physical hardware. We show that the proposed numerical scheme converges in law to a solution that is defined in the sense of stochastic variational inequalities (SVIs). Under strengthened assumptions the convergence can be show to holds even in probability. As a by product of our convergence analysis we provide a generalization of the concept of probabilistically weak solutions of stochastic partial differential equation (SPDEs) to the setting of SVIs. We also prove convergence of the numerical scheme to a probabilistically strong solution in probability if pathwise uniqueness holds. We perform numerical simulations to illustrate the behavior of the proposed numerical scheme as well as its non-conforming variant in the context of image denoising.

Acknowledgment
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Nonparametric Diffusivity Estimation for the Stochastic Heat Equation from Noisy Observations

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Abstract

We estimate the spatially varying diffusivity of a stochastic heat equation from observations perturbed by additional static noise. To that end, we employ a two-step localization procedure, more precisely, we combine local state estimates into a locally linear regression approach. Our analysis relies on quantitative Trotter–Kato type approximation results for the heat semigroup that are of independent interest. The presence of static noise leads to non-standard scaling behaviour of the model.

Acknowledgment

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Abstract sewing lemma

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Abstract

The sewing lemma is a tool that establishes the existence and the uniqueness of an integral given sufficiently smooth local approximations. Originally, the sewing lemma was stated for local approximations of Hölder-type continuity, see Gubinelli [3] and Feyel and de La Pradelle [1]. Recently, a Sobolev-type and Besov-type variants were shown by Liu, Prömel and Teichmann [4] and Friz, Seeger and Zorin-Kranich [2].
In the talk, we discuss an abstract version of the sewing lemma in general function spaces. The abstract formulation allows us to concentrate on the common property of the spaces where the sewing lemma works well without the technicalities of the particular function spaces. We identify the common property to be the behaviour of the increments of the local approximations under certain time dilations. Moreover, all of the results mentioned above follow from the abstract result as direct corollaries.

Acknowledgment

The research was supported by the Czech Science Foundation project no. 22-12790S.

References

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[2] Friz, P. K., Seeger, B., Zorin-Kranich, P., (2022): Besov rough path analysis. *J. Differential Equations* **339**, 152–231.

[3] Gubinelli, M., (2004): Controlling rough paths. *J. Funct. Anal.* **216**(1), 86–140.

[4] Liu, C., Prömel, D. J., Teichmann, J., (2021): On Sobolev rough paths. *J. Math. Anal. Appl.* **497**(1), Paper No. 124876.

Filtering SPDEs with spatio-temporal point process observations

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Abstract

In this joint talk we present a mathematical framework for filtering problems arising from biophysical applications where data is collected from confocal laser scanning microscopy recordings of the space-time evolution of intracellular wave dynamics of biophysical quantities. In these applications, signals are described by stochastic partial differential equations (SPDEs) and observations can be modelled as functionals of marked point processes whose intensities depend on the underlying signal. We derive both the unnormalized and normalized filtering equations for these systems, demonstrate the asymptotic consistency and approximations of finite dimensional observation schemes respectively partial observations. Our theoretical results are validated through extensive simulations using synthetic and real data.

In a broader statistical context, this is one component of a statistical toolbox for the quantitative model evaluation of SPDEs modeling space-time evolution of biophysical quantities on the intracellular level, that has to be combined subsequently with statistical inference methods for parameter estimation.

Non-parameteric estimation for linear SPDEs on arbitrary domains based on discrete observations

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Abstract

Most statistical methods for stochastic partial differential equations (SPDEs) based on discrete observations are limited to one space dimension or to quite restrictive settings. In order to study SPDEs on a bounded domain driven a stochastic noise process which is white in time and possibly colored in space, we aim for bridging the gap between two popular observations schemes studied for statistics for SPDEs, namely, discrete observations and local measurements. To this end, we have to extend the local measurements approach to kernels of distribution type. This link allows us to construct a non-parametric estimator for the diffusivity based on discrete observations.

Multidimensional Stein-Malliavin calculus and asymptotic independence

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Abstract

We develop an extension of the Stein-Malliavin calculus which allows to measure the Wasserstein distance between the probability distributions of (X, Y) and (Z, Y), where X, Y are arbitrary random vectors and $Z \sim N(0, \sigma^2)$ is independent of Y. We apply this method to study the asymptotic independence for sequences of random vectors with a particular focus on the case of some estimators for some parameters of SPDEs.

Acknowledgment

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References

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[2] Tudor, C. A., Zurcher, J., (2024+): Multidimensional Stein-Malliavin calculus for the multivariate Gaussian distribution. *In progress.*

Parameter estimation in hyperbolic linear SPDEs from multiple measurements

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Abstract

The coefficients of elastic and dissipative operators in a linear hyperbolic SPDE are jointly estimated using multiple spatially localised measurements. As the resolution level of the observations tends to zero, we establish the asymptotic normality of an augmented maximum likelihood estimator. The rate of convergence for the dissipative coefficients matches rates in related parabolic problems, whereas the rate for the elastic parameters also depends on the magnitude of the damping. The analysis of the observed Fisher information matrix relies upon the asymptotic behaviour of rescaled M, N-functions generalising the operator sine and cosine families appearing in the undamped wave equation.

References

[1] Tiepner, A., Ziebell, E., (2024): Parameter estimation in hyperbolic linear SPDEs from multiple measurements. Preprint, arXiv:2404.18823.